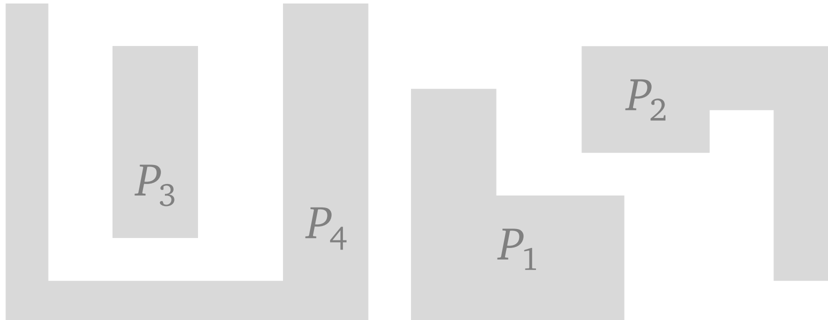
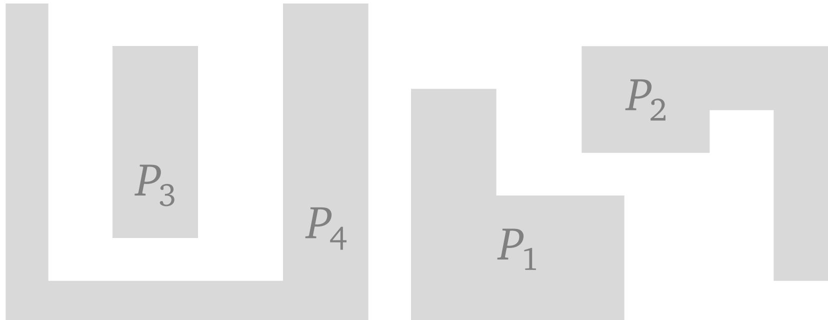


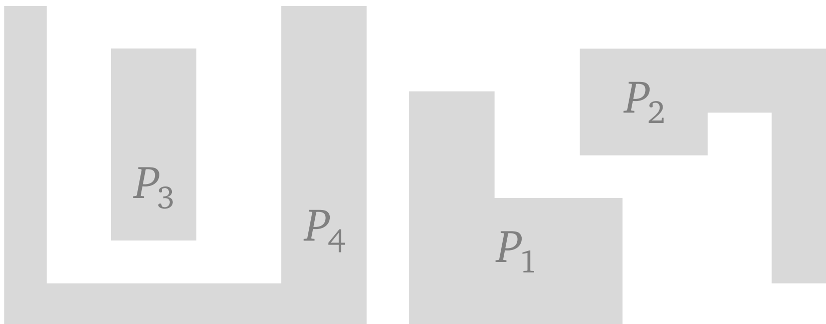
Touring a Sequence of Orthogonal Polygons

Katrin Casel Sándor Kisfaludi-Bak Linda Kleist
Jeroen S.K. Lamme Eunjin Oh Yanheng Wang



Disjoint orthogonal polygons $P_1, \ldots, P_k$ (order is given)

Total number of vertices $= n$



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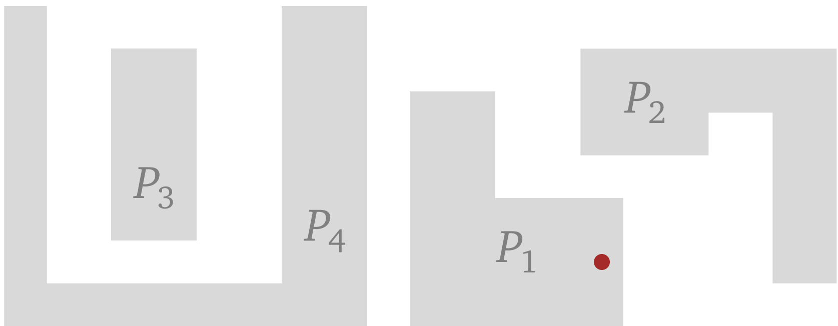
Total number of vertices $= n$

Pick $p_i \in P_i$ to minimize $\|p_1 - p_2\| + \|p_2 - p_3\| + \cdots + \|p_{k-1} - p_k\|$



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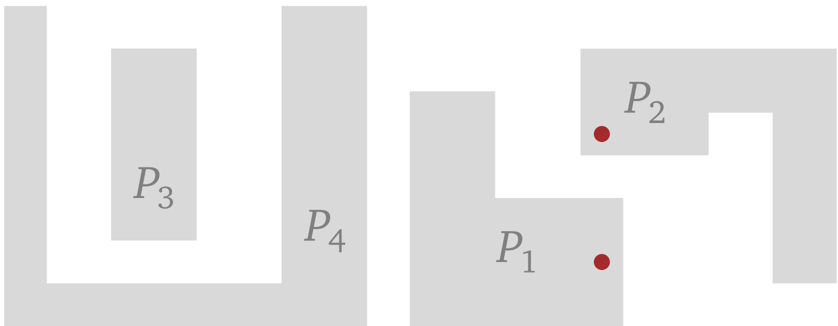

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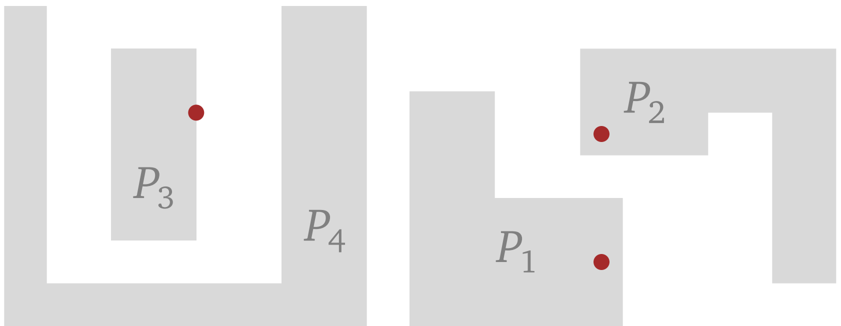

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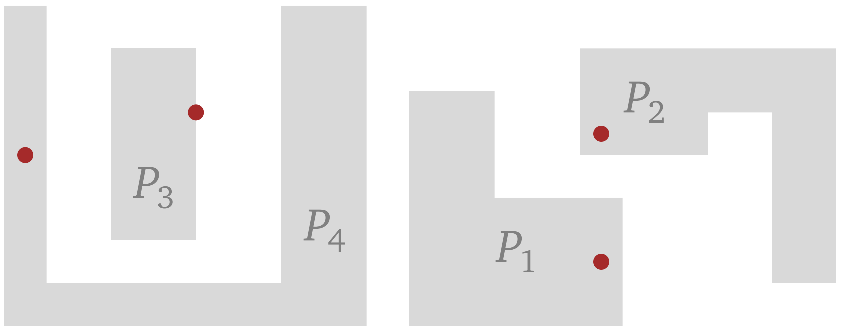

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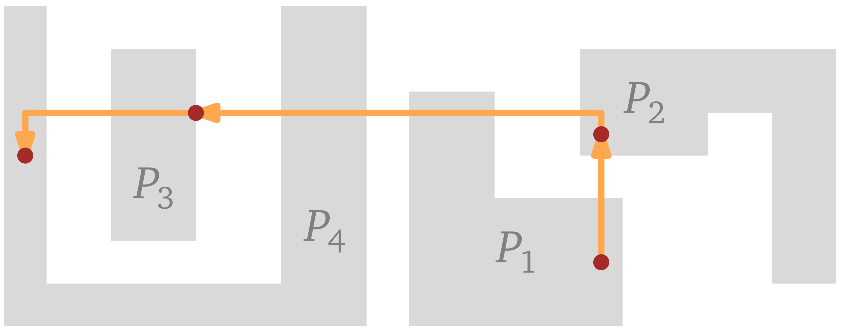

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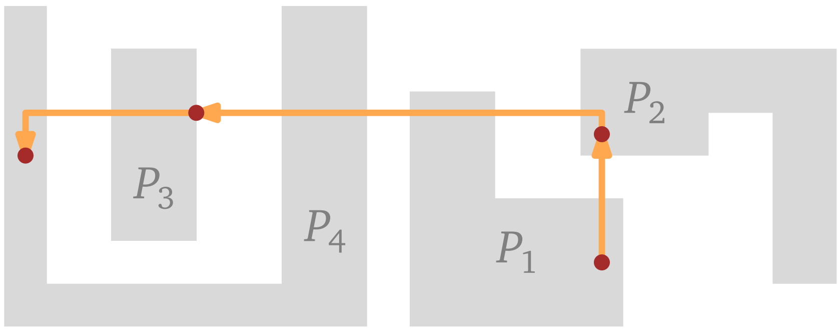

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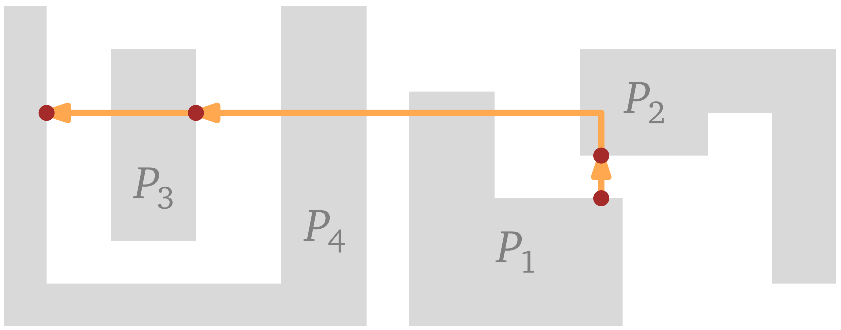

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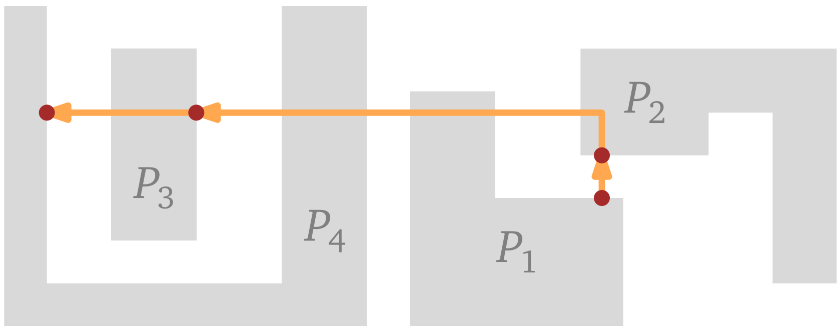

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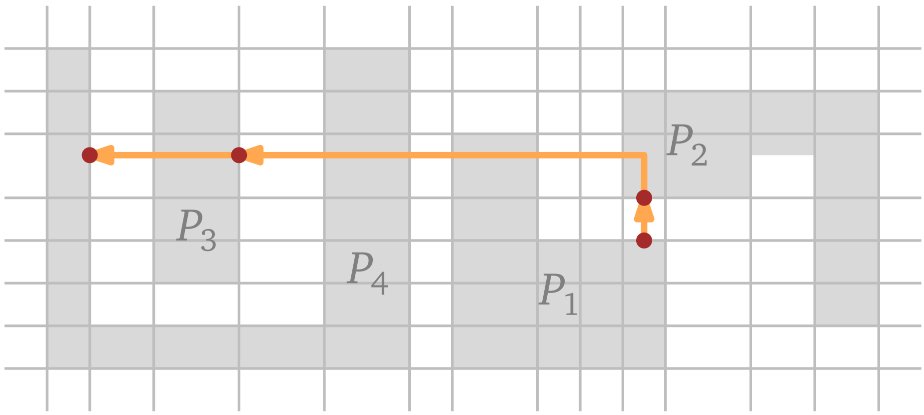

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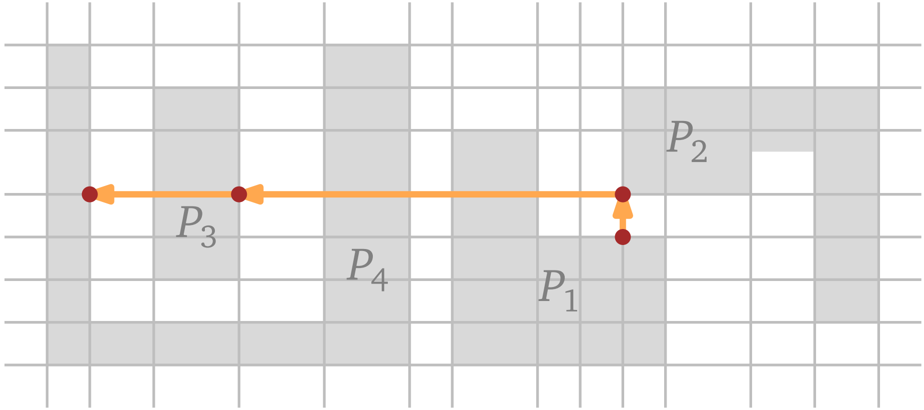

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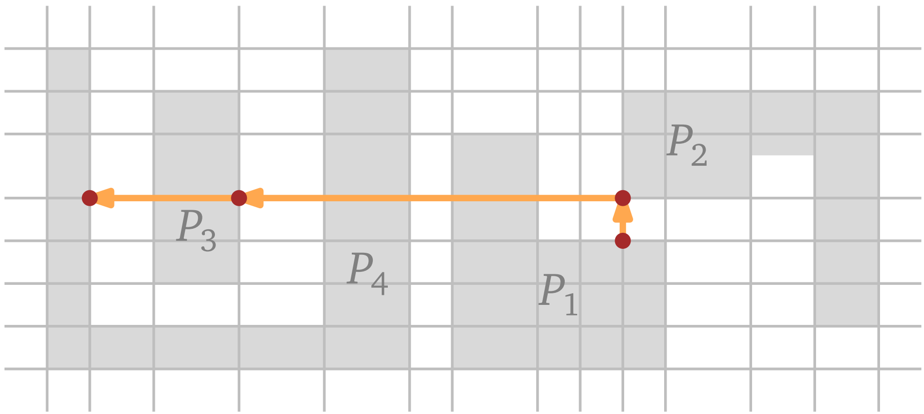

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$k :=$ #polygons
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Can we break the quadratic barrier?
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– Case $s \leq n^{0.8}$
– Case $s > n^{0.8}$ (and thus $k < n^{0.2}$)

Case $s \leq n^{0.8}$

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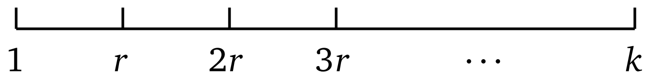
Idea: time localization



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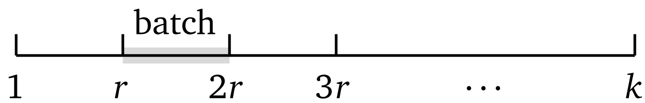

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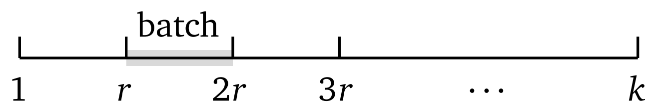

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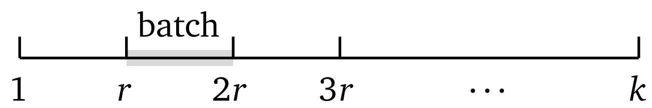

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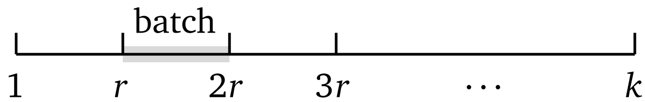

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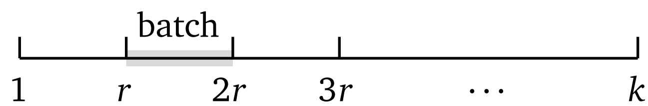

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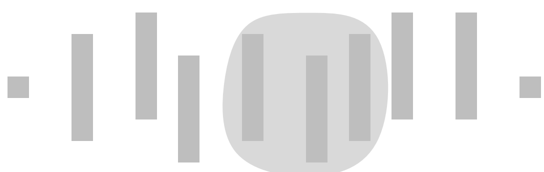

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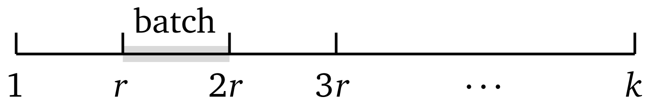

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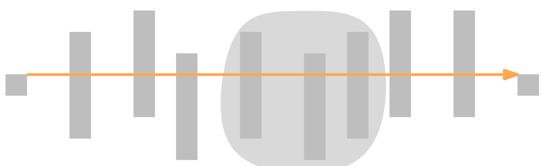

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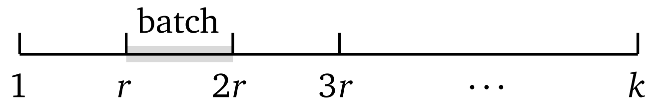

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Lemma. Fix $a = jr$ and $p \in \text{portal}_{a+r}$. There is a shortest path visiting $P_1, \ldots, P_{a+r}, p$ with one of the following properties:

- it visits $\text{portal}_1, \ldots, \text{portal}_{a-1}, \text{terminal}_a, \ldots, \text{terminal}_{a+r}, p$;
- it visits $\text{portal}_1, \ldots, \text{portal}_{a-1}$ and is horizontal afterwards;
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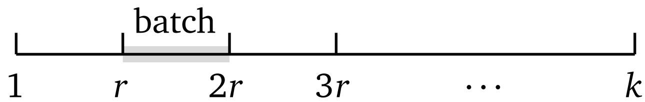

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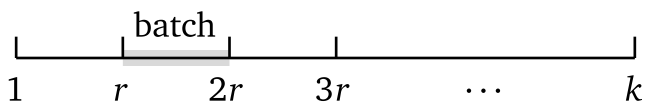

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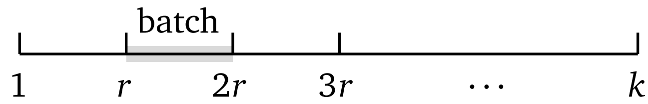

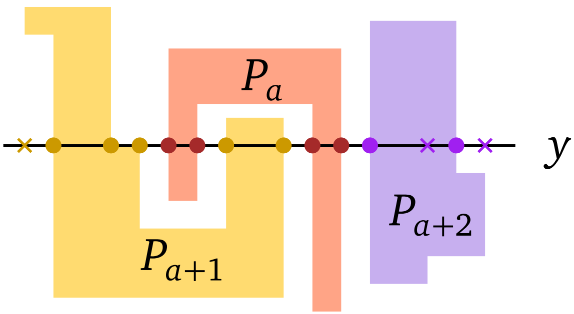

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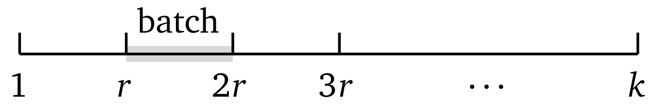

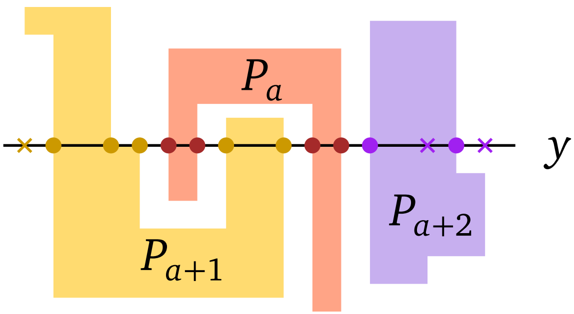

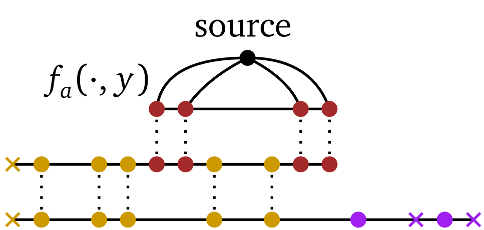

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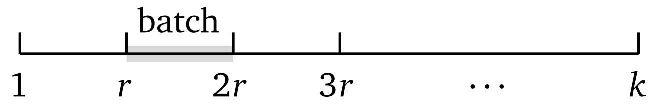

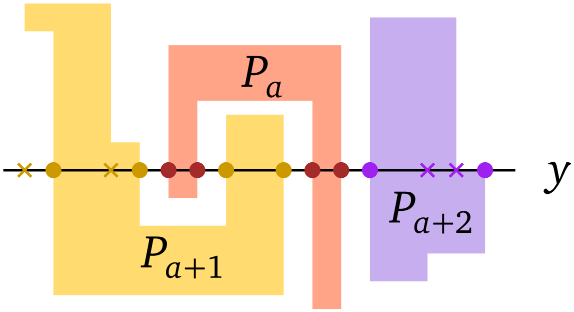

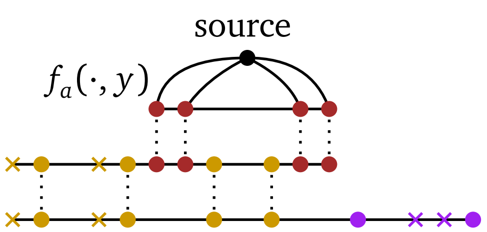

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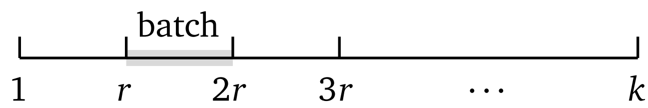
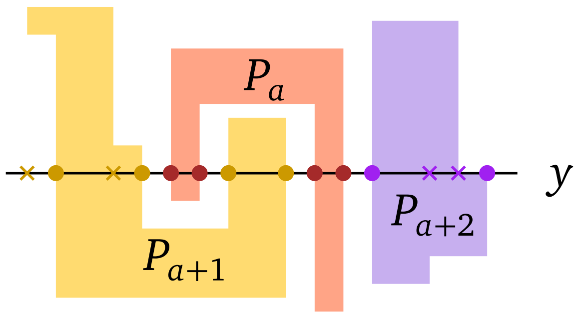
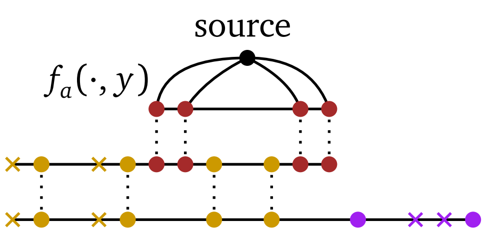
Idea: time localization





Sweep the line for n steps



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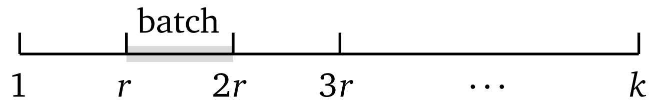

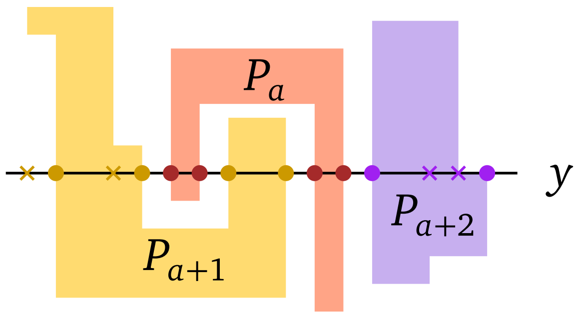

Sweep the line for n steps

#insertions/deletions $= O(rs)$

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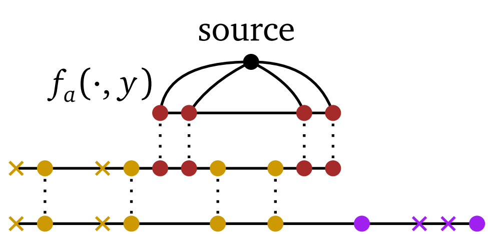
#queries $= O(ns)$



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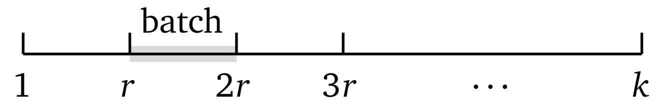

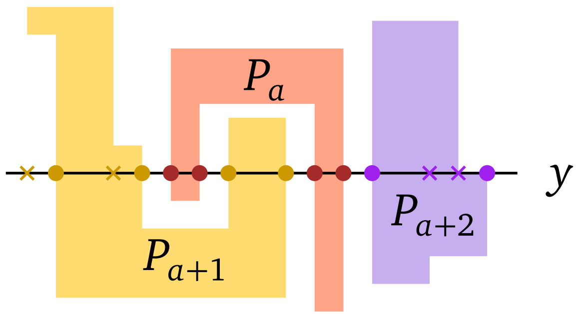

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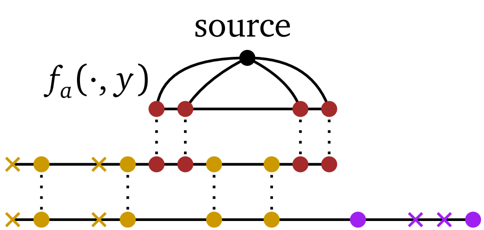

Dynamic planar SSSP

[Das, Probst Gutenberg, Wulff-Nilsen 2022]

update time $\widetilde{O}(N^{1/2})$

query time $\widetilde{O}(N^{1/2})$

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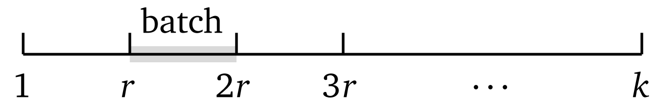

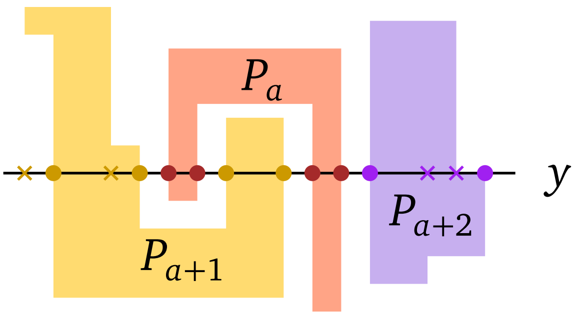

Sweep the line for n steps

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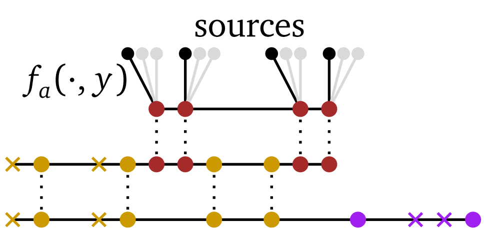

Dynamic planar MSSP

[Charalampopoulos, Karczmarz 2020]

update time $\widetilde{O}(N^{4/5} + N^{3/4}s^{1/4})$

query time $\widetilde{O}(1)$

Case $s \leq n^{0.8}$

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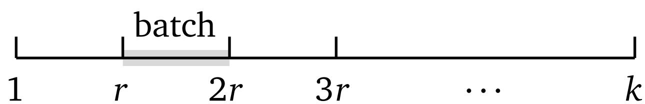

Running time:

$$\text{each batch } \widetilde{O}\Big(\underbrace{(rs)^2}_{\text{DP}} + \underbrace{n^{9/5} + n^{7/4}s^{1/4} + ns}_{\text{line sweep}}\Big)$$

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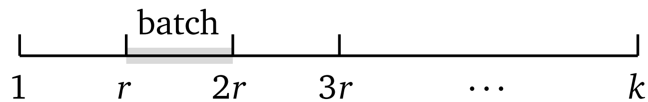

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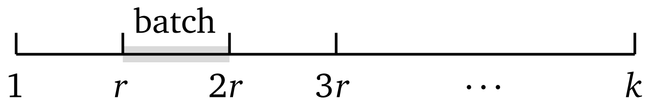

Running time:

each batch $\widetilde{O}\Big(\underbrace{(rs)^2}_{\text{DP}} + \underbrace{n^{9/5} + n^{7/4}s^{1/4} + ns}_{\text{line sweep}}\Big) \leq \widetilde{O}(n^{1.96})$

choose $r := n^{0.98}/s$

overall $\widetilde{O}(n^{1.96}) \cdot n^{0.02} = \widetilde{O}(n^{1.98})$

Case $s > n^{0.8}$

$k :=$ #polygons
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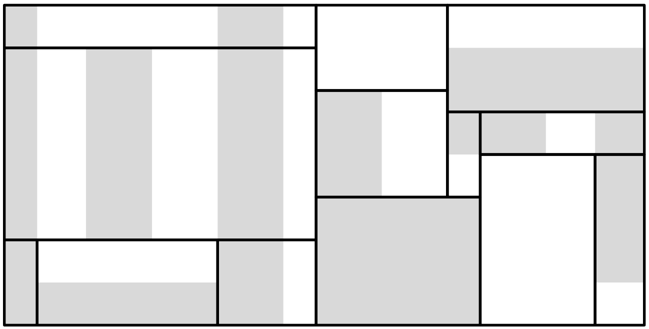
Idea: space localization



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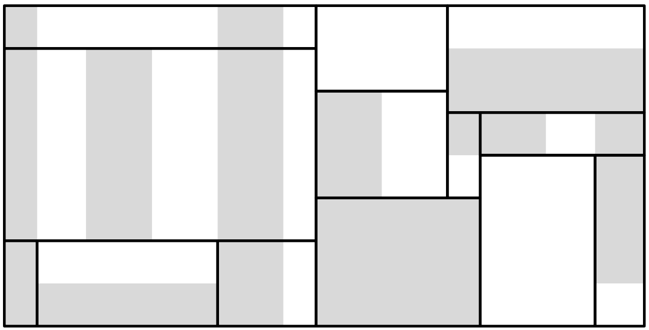

rectangle partition with small stabbing number

[De Berg, Van Krevald 1993]

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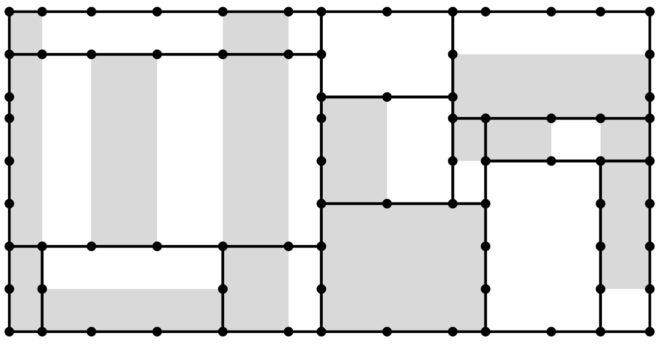

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"hubs"

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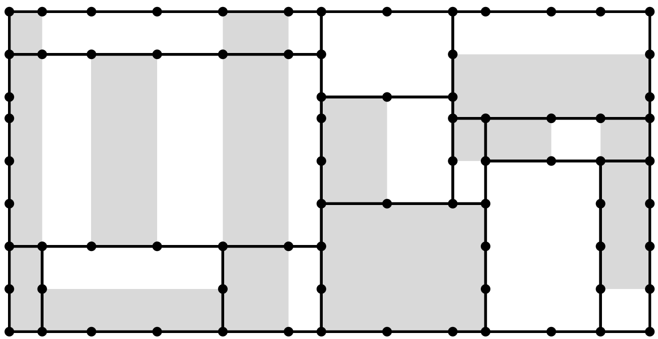

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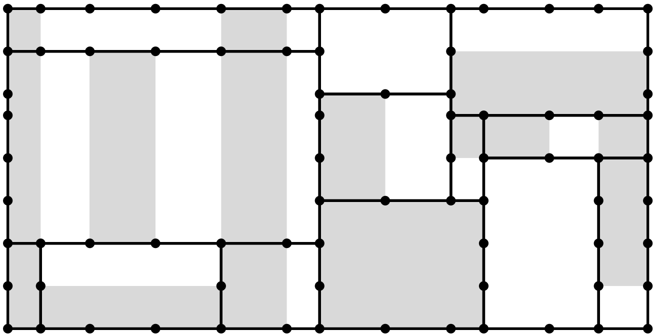

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precompute costs inside rectangles, then DP on hubs

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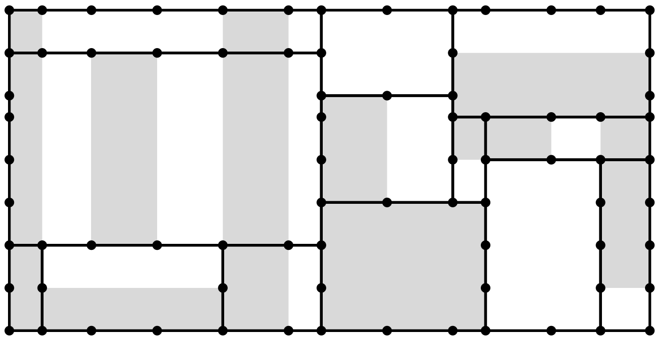

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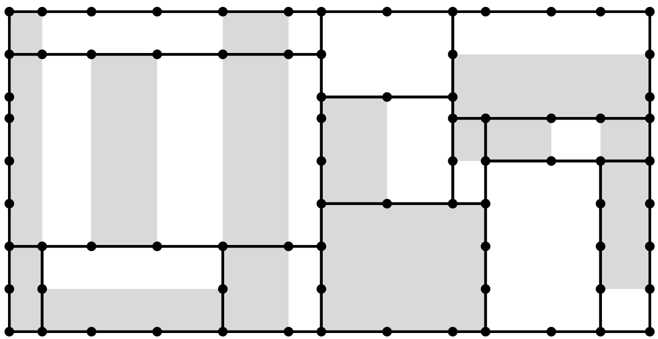

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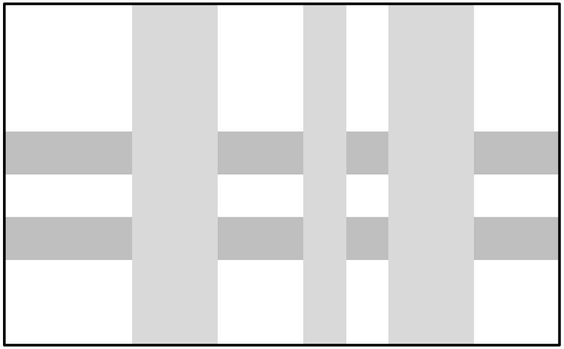

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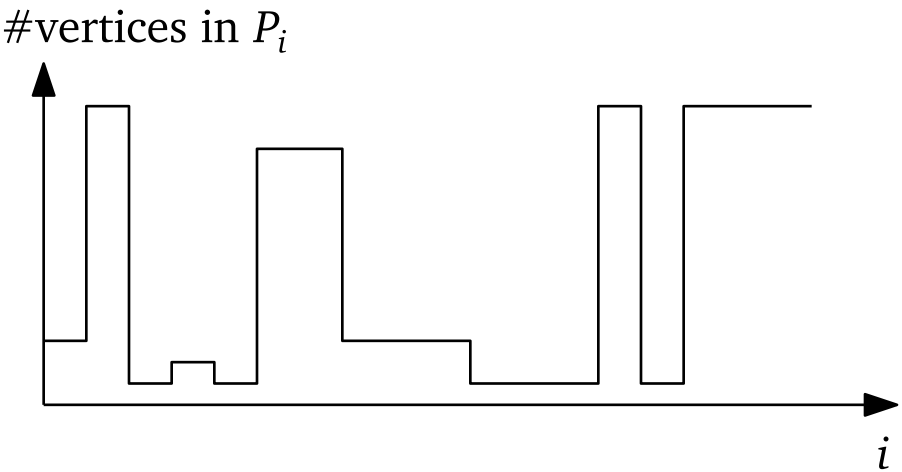

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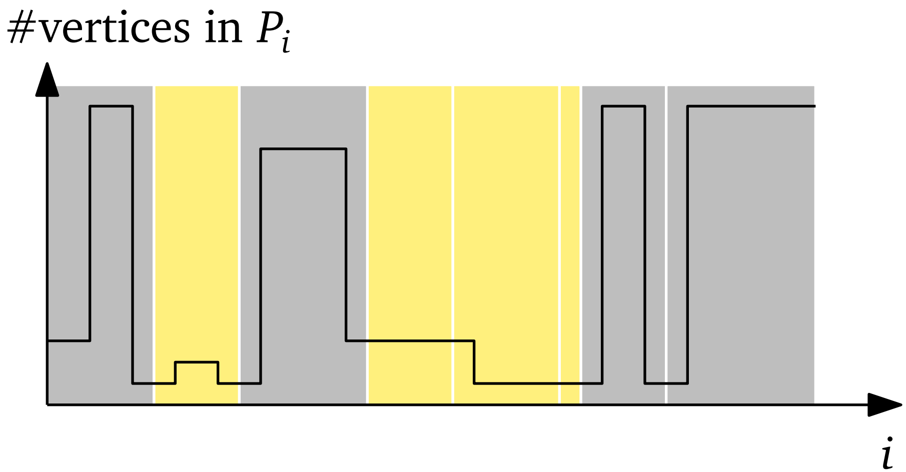

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step-disjoint varying

Open Problems

Intersecting polygons?

Faster dynamic planar MSSP?

Superlinear lower bound?

Euclidean distance?

$\widetilde{O}(n^2)$ time for disjoint convex polygons [Dror, Efrat, Lubiw, Mitchell 2003]