

Markov Chain Toolbox

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Distances and inner products

$$\|\mu - \nu\| := \frac{1}{2} \sum_{x \in \mathcal{X}} |\mu(x) - \nu(x)| = \max_{S \subseteq \mathcal{X}} |\mu(S) - \nu(S)|$$

$$\|f\|_p := \left(\sum_x |f(x)|^p \right)^{1/p}$$

$$\|f\|_1 = 2\|f\|$$

$$\|f\|_2 = \sqrt{\langle f, f \rangle}$$

$$\|f\|_2 = \sqrt{\langle f, f \rangle_\pi}$$

$$\|f\|_\infty = |f|_\infty = \max_x |f(x)|$$

$$\langle f, g \rangle := \sum_x f(x)g(x)$$

$$\langle f, g \rangle_\pi := \sum_x f(x)g(x)\pi(x)$$

$$\langle f, \mathbf{1} \rangle_\pi = E_\pi(f)$$

$$\langle f_i, \mathbf{1} \rangle_\pi = 0$$

Eigenvalues and eigenfunctions

$$\{f_i\}, \{\lambda_i\}$$

$$\lambda_1 = 1 > \lambda_2 \geq \dots \geq \lambda_{|\mathcal{X}|} \geq -1$$

$$\gamma_* := 1 - \max_{i>1} |\lambda_i|$$

$$\gamma := 1 - \lambda_2$$

$$t_{\text{rel}} := 1/\gamma_*$$

Other parameters

$$Q(x, y) := \pi(x)P(x, y)$$

$$\Phi(S) := \frac{\sum_{x \in S, y \notin S} Q(x, y)}{\pi(S)}$$

$$B(e) := \frac{1}{Q(e)} \sum_{xy \in E: e \in \Gamma_{xy}} Q(x, y) |\Gamma_{xy}|$$

$$\mathcal{E}(f) := \langle (I - P)f, f \rangle_\pi = \frac{1}{2} \sum_{x, y} (f(x) - f(y))^2 Q(x, y)$$

